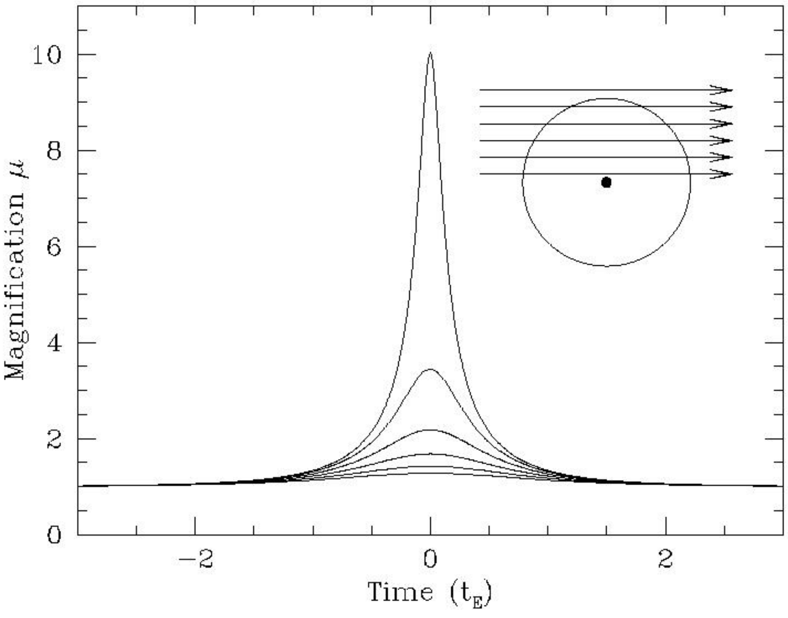
Microlensing

- **Microlensing: When a Foreground Lensing Mass (Star or Remnant) Passes In Front of a Background Star**
  - **Brightness Magnification Changes Result in a Predictable Light Curve (0.3 mags or more)**
  - **Distinguishable Light Curve, Independent of λ**
  - **Low Probability But Statistics Provides Constraint on Space Density of Lenses**
  - **Successful Survey Requires Large Number of Background Sources (~ 10^6)**
- **Light Curve**
  - **Brightness Depends on Einstein Radius and Impact Parameter**
  - **Timescale Depends on Transverse Velocity:**

$$t_0 = \frac{D_d \theta_E}{v}$$

$$= 0.214\, yr \left(\frac{M}{M_\odot}\right)^{1/2} \left(\frac{D_d}{10kpc}\right)^{1/2} \times \left(\frac{D_{ds}}{D_s}\right)^{1/2} \left(\frac{200km/s}{v}\right)$$

  - **Note that the timescale is degenerate in mass, distances and transverse velocity and the full light curve only results in two constraints (t_0 and Δy).**



Brightness magnification of a source due to a point mass for different paths relative to the Einstein ring (Wambsganss 2006).

Microlensing - II

- **Optical Depth for Lensing Depends on Lens Volume Density and Cross-section (Einstein Radius)**

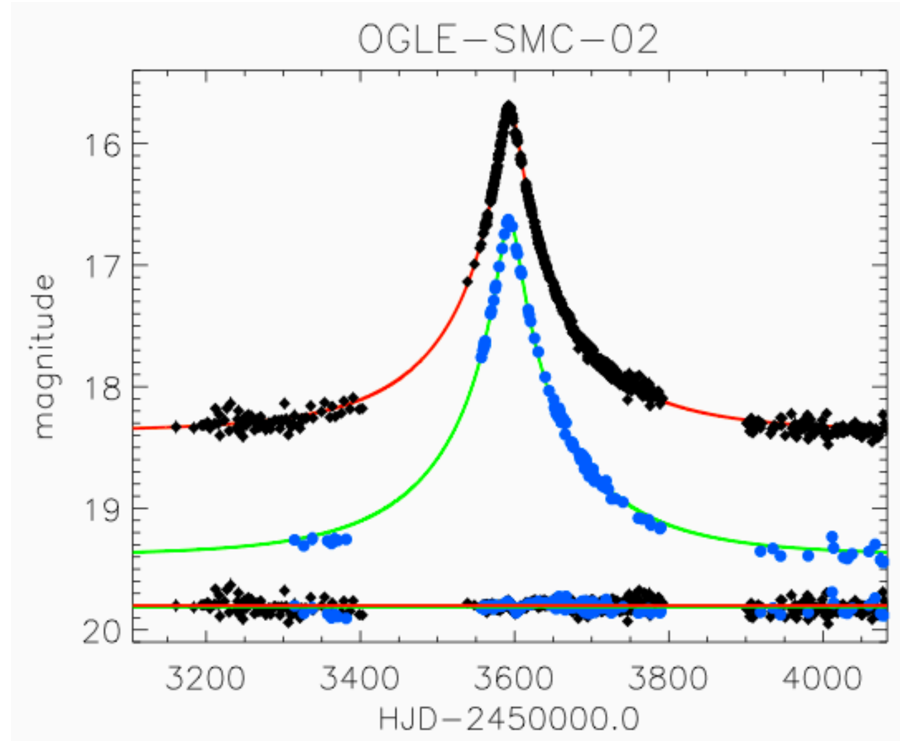
$$\tau = \frac{1}{\delta\omega}\int dVn(D_d)\pi\theta_E^2$$

Where $dV = \delta\omega D_d^2 dD_d$ is the volume of an infinitesimal spherical shell with radius D_d with solid angle δω. Thus:

$$\tau = \int_0^{D_s} \frac{4\pi G\rho}{c^2}\frac{D_d D_{ds}}{D_s} dD_d$$
$$= \frac{4\pi G}{c^2} D_s^2 \int_0^1 \rho(x)x(1-x)dx$$

With $x \equiv {}^{D_d}/_{D_s}$ and ρ being the mass density of lenses (MACHOs, etc.). Note that is r is constant along the line of sight this simplifies to:

$$\tau = \frac{2\pi}{3}\frac{G\rho}{c^2}D_s^2$$



Light curve for a microlensing event from the OGLE survey of the Small Magellanic Cloud

More Realistic Potentials -I

- **Real Galaxies and Clusters of Galaxies Lack Spherical Symmetry**
- **Realistic Models Can Be Constructed From 3d Density Distributions Like a Softened Isothermal with a Core. The Projected Mass Density is:**

$$\boldsymbol{\Sigma(\theta_1, \theta_2) = \frac{\Sigma_0}{\left[\theta_c^2 + (1-\epsilon)\theta_1^2 + (1+\epsilon)\theta_2^2\right]^{1/2}}}$$

The potential corresponding to this mass distribution is complicated but for vanishing core radius the deflection angle and magnification are relatively simple:

$$\boldsymbol{\alpha_1 = \frac{8\pi G\Sigma_0}{\sqrt{2\epsilon c^2}} \tan^{-1}\left[\frac{\sqrt{2\epsilon}\cos\phi}{(1-\epsilon\cos 2\phi)^{1/2}}\right],}$$

$$\boldsymbol{\alpha_2 = \frac{8\pi G\Sigma_0}{\sqrt{2\epsilon c^2}} \tan^{-1}\left[\frac{\sqrt{2\epsilon}\sin\phi}{(1-\epsilon\cos 2\phi)^{1/2}}\right],}$$

$$\boldsymbol{\mu^{-1} = 1 - \frac{8\pi G\Sigma_0}{c^2\left(\theta_1^2 + \theta_2^2\right)^{1/2}(1-\epsilon\cos 2\phi)^{1/2}}}$$

More Realistic Potentials - II

- **A Simpler Approach is to Specify an Elliptical Effective Lensing Potential:**

$$\psi(\theta_1,\theta_2) = \frac{D_{ds}}{D_s} 4\pi \frac{\sigma_v^2}{c^2} \left[\theta_c^2 + (1-\epsilon)\theta_1^2 + (1+\epsilon)\theta_2^2\right]^{1/2}$$

At large ε the contours become unrealistically peanut shaped, but this parameterization works very well for small ε.

- **Lensing Masses are Seldom Isolated. They Are Often Found in Galaxy Groups or Clusters. Thus We Can Add External Convergence and Shear to Account for Large Scale Structure. Specifically We Add:**

$$\psi(\theta_1,\theta_2) = \frac{\kappa}{2}(\theta_1^2+\theta_2^2) + \frac{\gamma}{2}(\theta_1^2-\theta_2^2)$$

In the coordinates of the external shear. This approach is often degenerate with adding ellipticity to the principle lensing mass. The determinant of the Jacobian can vanish (mag → ∞) at critical curves in the lens plane which correspond to caustic curves in the source plane.

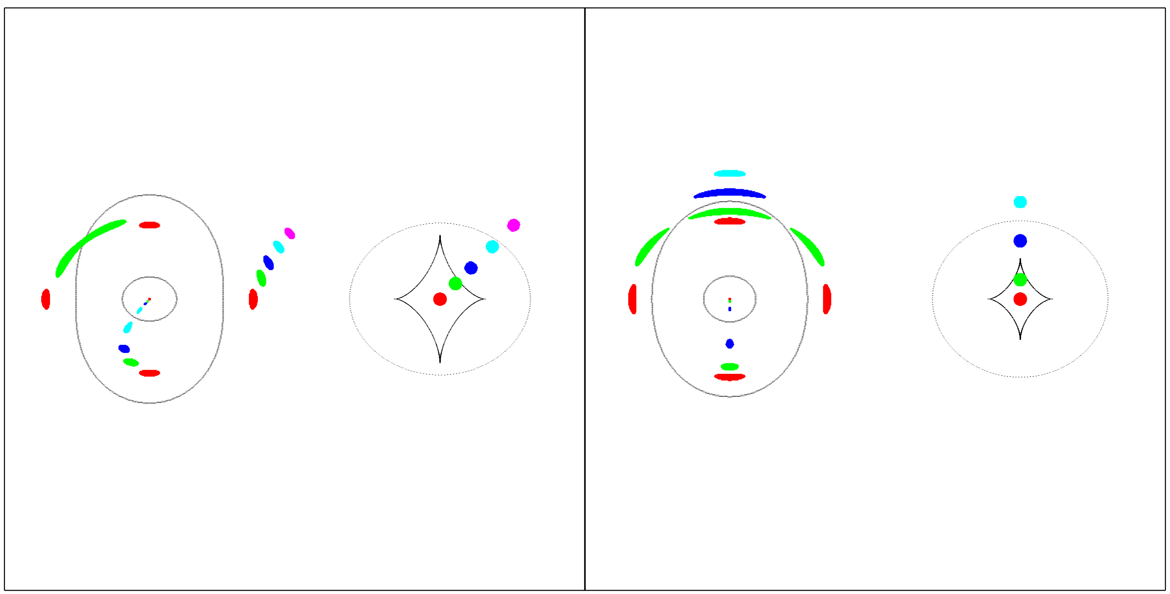


Image morphologies produced by models of a compact source (colored dots) moving w.r.t. an elliptical lensing potential. Left: the source is crossing a fold caustic. Right: the source is crossing a cusp caustic. The left side of each panel shows the image plane and the right shows the source plane.

References

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