Weak Gravitational Lensing

- **Foreground Masses Can Magnify and Distort Background Sources**
- **Faint, High-z Galaxies Provide a "Cosmic Wallpaper" Upon Which the Lensing Mass is Projected**
  - **At even modest redshifts there are ~ 50 galaxies/arcmin^2. The deepest surveys with HST reveal ~ 100/arcmin^2.**
- **Source Sizes Are Unknown but We can Compute an Average Shape for Un-lensed Field Samples and Assume Random Orientations**
- **For Simplicity Let's Assume Average Shape is Circular with Radius θ. For a Weakly Lensed Background Source:**

$$\theta_\perp = \frac{\theta}{1-\kappa-\gamma}\ ,\theta_\parallel = \frac{\theta}{1-\kappa+\gamma}$$

Where $\perp$ *or* $\parallel$ is relative to the vector between the lensing mass and the background source. We define the ellipticity as:

$$\varepsilon = \frac{a-b}{a+b} = \frac{2\gamma}{2(1-\kappa)} = \frac{\gamma}{1-\kappa} \approx \gamma$$

So the "excess ellipticity" of a background source is a measure of the shear. Now consider a source with an ellipticity (magnitude and position angle):

$$\epsilon_1 = \epsilon \cos 2\phi, \qquad \epsilon_2 = \epsilon \sin 2\phi$$

If the intrinsic source ellipticity is ϵ_i^s then:

$$\epsilon_i = \epsilon_i^s + \gamma_i = \epsilon^s \begin{pmatrix} \cos 2\phi \\ \sin 2\phi \end{pmatrix} + \begin{pmatrix} \gamma_1 \\ \gamma_2 \end{pmatrix}$$

But after averaging in annular bins over many sources the first term disappears such that:

$$\langle \epsilon \rangle_\theta = \langle \gamma \rangle_\theta$$

So if we have a measure of the shear field [γ(θ)], how do we turn this into a mass distribution for the cluster?

Cluster Masses from Weak Lensing

- Answer Lies with Fourier Transforms (Kaiser & Squires 1993)

$$\kappa = \frac{1}{2}(\psi_{11} + \psi_{22}) \Rightarrow \hat{\kappa} = -\frac{1}{2}(k_1^2 + k_2^2)\hat{\psi}$$

$$\gamma_1 = \frac{1}{2}(\psi_{11} - \psi_{22}) \Rightarrow \widehat{\gamma_1} = -\frac{1}{2}(k_1^2 - k_2^2)\hat{\psi}$$

$$\gamma_2 = \psi_{12} \Rightarrow \widehat{\gamma_2} = -k_1 k_2 \hat{\psi}$$

Where k is a wave vector conjugate to the angular position vector θ. We can now eliminate ψ. Noting that:

$\left[k^{-2}\begin{pmatrix} k_1 - k_2 \\ 2k_1k_2 \end{pmatrix}\right]\left[k^{-2}\begin{pmatrix} k_1^2 - k_2^2 & 2k_1k_2 \end{pmatrix}\right] = 1$, and that

$\begin{pmatrix} \widehat{\gamma_1} \\ \widehat{\gamma_2} \end{pmatrix} = k^{-2}\begin{pmatrix} k_1^2 - k_2^2 \\ 2k_1k_2 \end{pmatrix}\hat{\kappa}$, then we find that:

$$\hat{\kappa} = k^{-2}\begin{pmatrix} k_1^2 - k_2^2 & 2k_1k_2 \end{pmatrix}\begin{pmatrix} \widehat{\gamma_1} \\ \widehat{\gamma_2} \end{pmatrix} = k^{-2}\left[(k_1^2 - k_2^2)\gamma_1 + 2k_1k_2\widehat{\gamma_2}\right]$$

But the convolution theorem: $(f \hat{*} g) = \hat{f}\hat{g}$ means that the convergence:

$$\kappa(\theta) = \frac{1}{\pi}\int d^2\,\theta'[D_1(\theta - \theta')\gamma_1 + D_2(\theta - \theta')\gamma_2]$$

With $D_1(\theta) = \frac{\theta_2^2 - \theta_1^2}{\theta^4}$ and $D_2(\theta) = \frac{2\theta_1\theta_2}{\theta^4}$. But it's not as easy as it looks. There are complications.

Cluster Masses from Weak Lensing - II

- **First, ellipticities actually measure the reduced shear:**

$$\langle g \rangle = \left\langle \frac{\gamma}{1-\kappa} \right\rangle$$

- **This is overcome by writing $\gamma = g(1-\kappa)$ and making use of the convolution theorem:**

$$\kappa(\theta) = \frac{1}{\pi} \int d^2\,\theta' [D_1(\theta-\theta')g_1(1-\kappa) + D_2(\theta-\theta')g_2(1-\kappa)]$$

Note that we need to be careful to ensure that our imaging field of view is large enough such that $\kappa \to 0$ at large θ in order to perform the transform.

- **Second, there is the problem of *<u>mass sheet degeneracy</u>*. Since the Jacobian can be multiplied with a factor λ: $A \to \lambda A \equiv A'$, without reflecting changes in ellipticity. That is, a transformation like:**

$1-\kappa' = \lambda(1-\kappa) \to \kappa' = 1-\lambda+\lambda\kappa$ cannot be detected. Maximum likelihood techniques can be applied. Namely we seek the potential that minimizes χ^2:

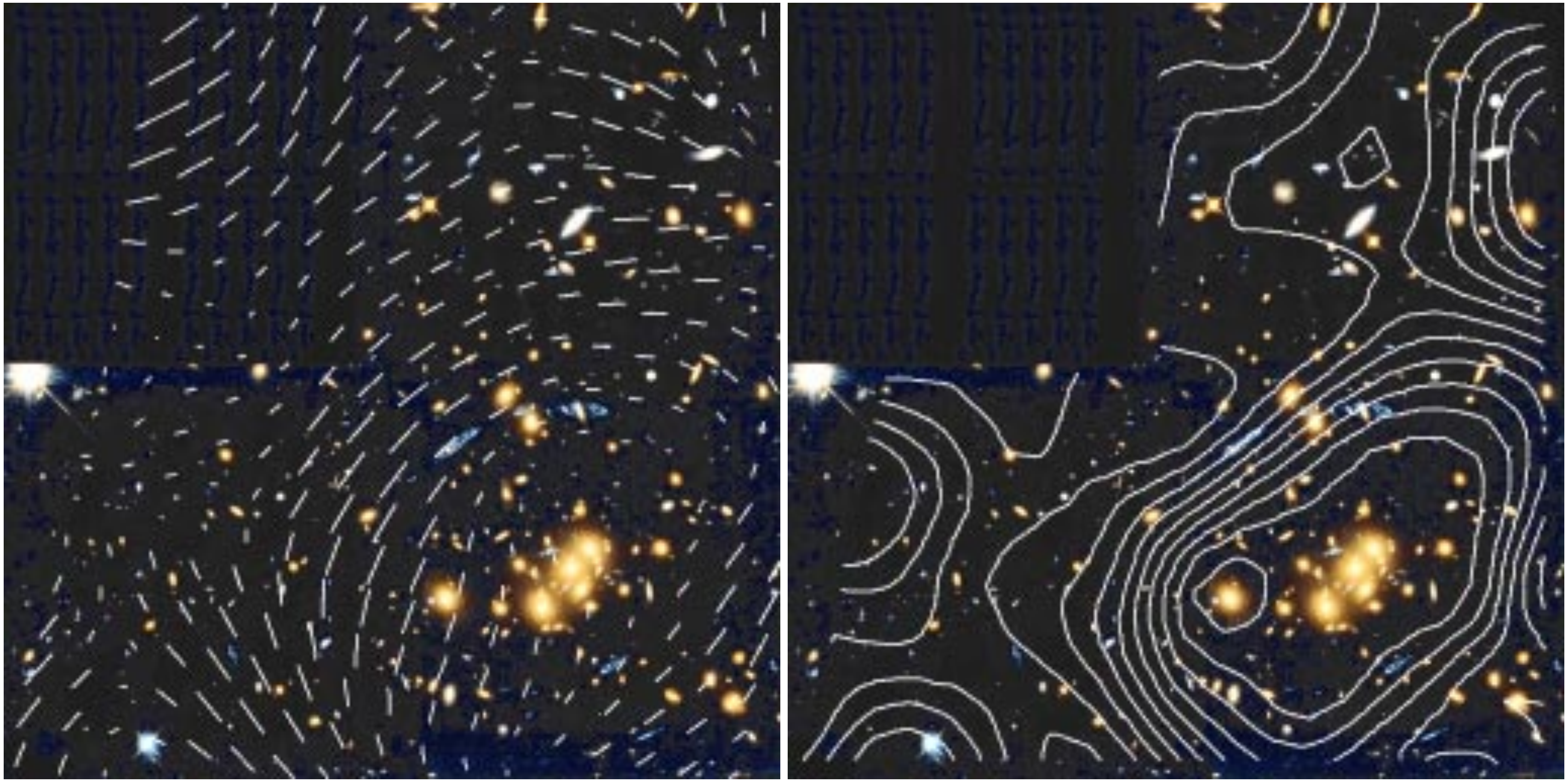
$$\chi^2 = \sum_{pixels} \frac{[\gamma_1 - \gamma_1(\psi)]^2 + [\gamma_2 - \gamma_2(\psi)]^2}{2\sigma_\gamma^2}$$

Where σ is the uncertainty in γ. We can avoid the mass sheet degeneracy by making use of calibration fields to determine the intrinsic size distribution of the source population and thus to enable κ to be constrained. That is, since:

$R \equiv \frac{1}{\mu} = (1-\kappa)^2 - \gamma^2 \approx 1 - 2\kappa$ and incorporating this into our minimization. For example:

$$\chi^2 = \sum_{pixels} \frac{[\gamma_1 - \gamma_1(\psi)]^2 + [\gamma_2 - \gamma_2(\psi)]^2}{2\sigma_\gamma^2} + \frac{[R - R(\psi)]^2}{2\sigma_R^2}$$

Example of Weak Lensing Mass Density Maps



Left: Weak lensing shear field measured at CFHT for the cluster CL0024 overlayed on the HST image of the central region (Mellier, Fort & Kneib 1993). Right: The reconstructed projected mass density (Seitz et al. 1996)

References

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