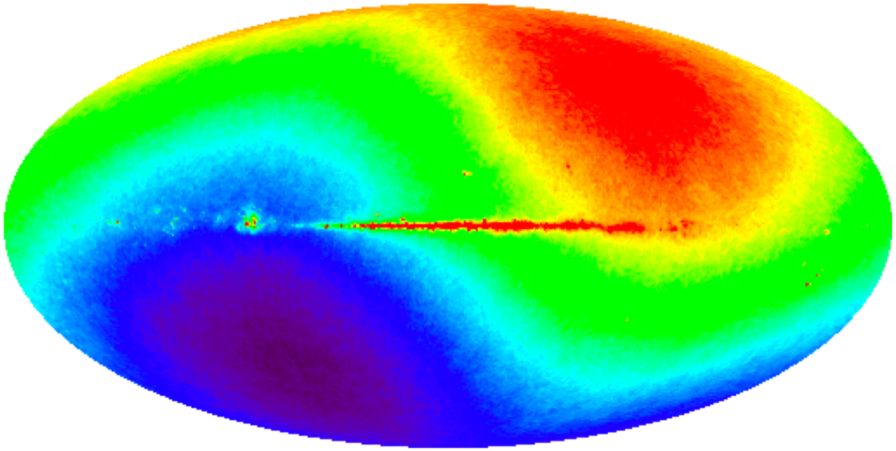
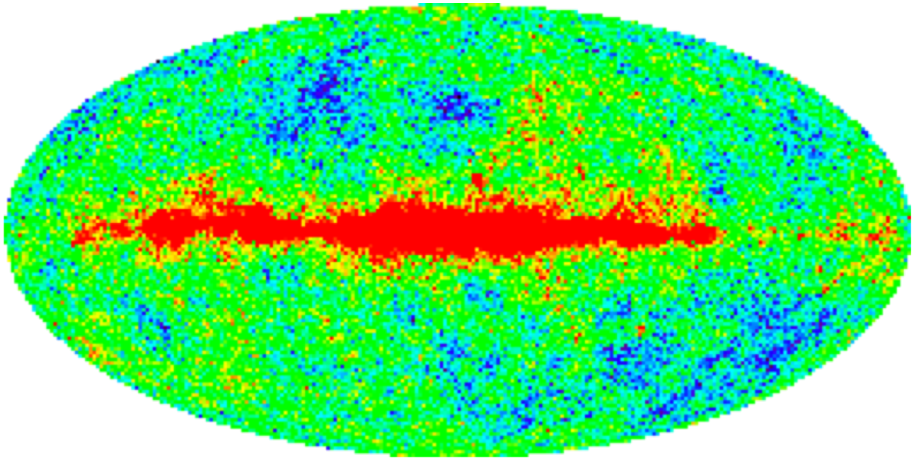
Astr 5465 April 30, 2018
Growth of Density Fluctuations

- Context:
- Fluctuations in $\frac{\delta T}{T} \approx 10^{-5}$ at z ~ 1000
- Fluctuations in $\frac{\delta \rho}{\rho} \approx 2$ today for superclusters
- So how do we get from the first to the second?





Recall the Freidman equation:

H^2-$\frac{8}{3}\pi G\rho = -\frac{k}{R^2} = 0$

A barely over-dense region evolves as a separate volume with slightly higher density:

H^2-$\frac{8}{3}\pi G\rho' = -\frac{k}{R^2}$ so we can describe the density contrast as the difference:

$-\frac{8}{3}\pi G(\rho - \rho') = -\frac{k}{R^2}$. or:

$\rho - \rho' = \frac{3k}{8\pi G R^2}$. So that $\frac{\delta\rho}{\rho} = \frac{3k}{8\pi G\rho R^2}$. Thus:

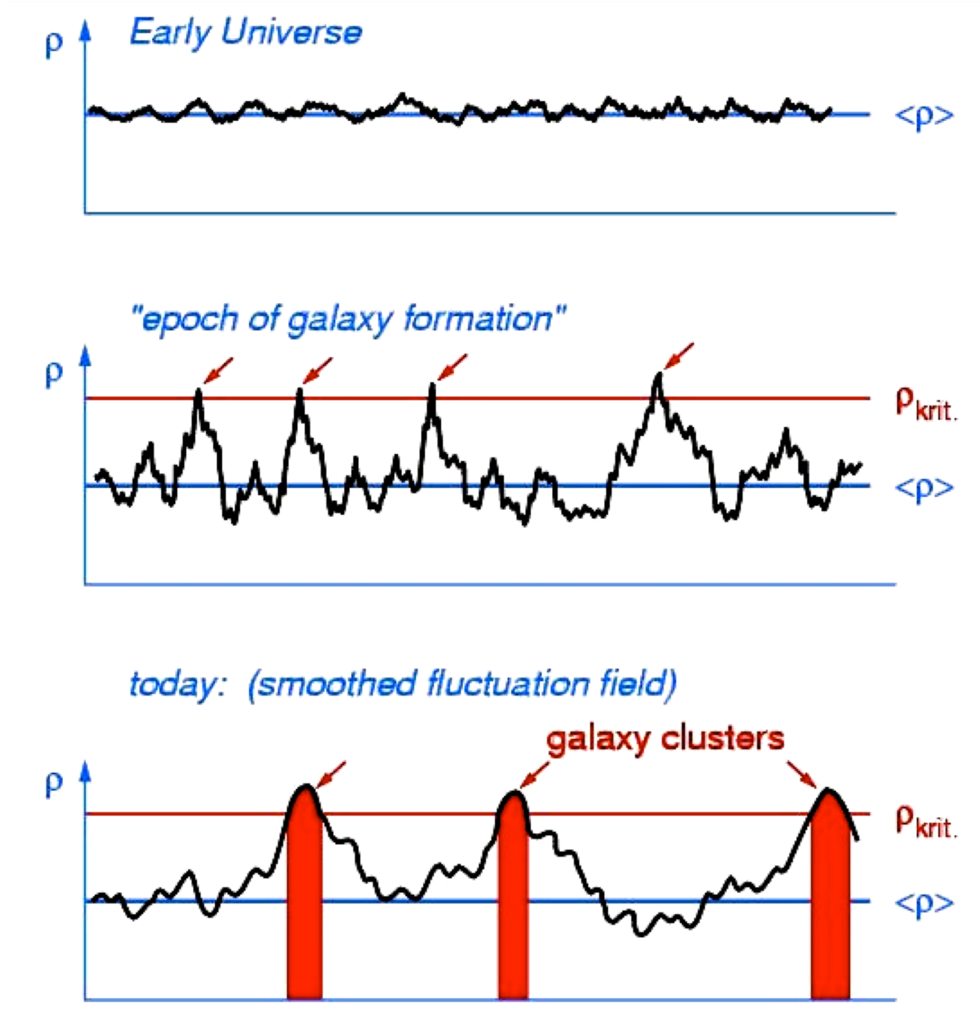
$\frac{\delta\rho}{\rho} \sim \frac{1}{\rho R^2} \sim \frac{1}{R^2 R^{-3}} \sim R \sim (1+z)^{-1}$ So we now see that

$$\left(\frac{\delta\rho}{\rho}\right)_0 = 10^{-5}\left(\frac{1+1000}{1+0}\right) \sim 10^{-2}$$

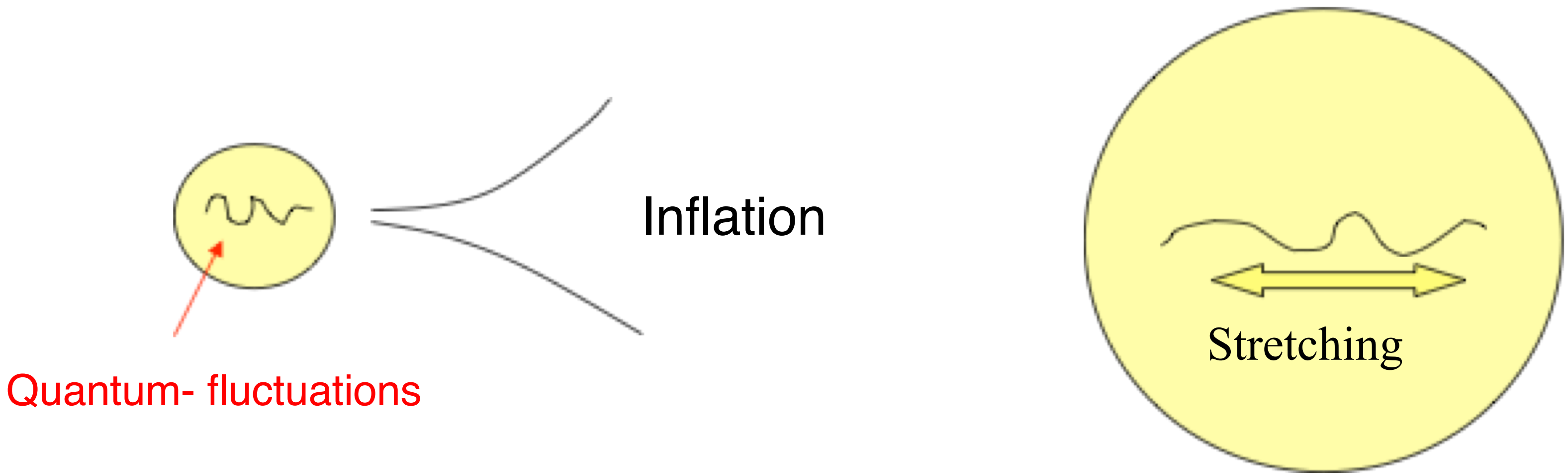
So, CMB fluctuations cannot grow into galaxies!

Growth of Density Perturbations

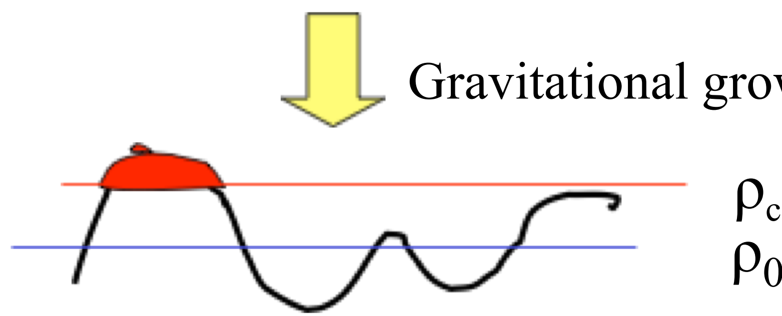
(based on a lecture by Hans Bohringer)



Gravitational growth of Perturbations

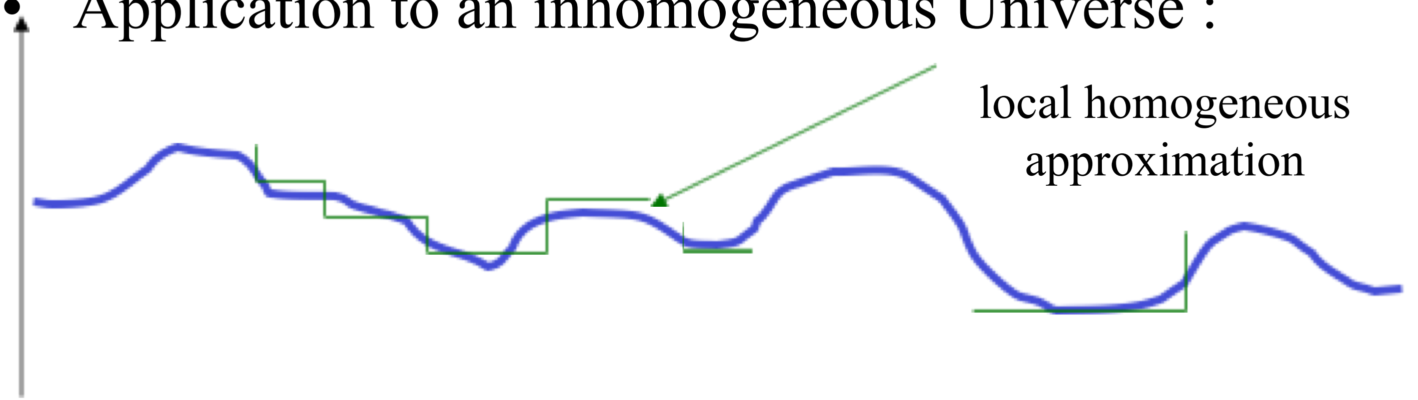


Quantum fluctuations are suddenly magnified to macroscopic scales during an inflationary phase in the early Universe and they are preserved by this magnification. Their amplitude grows subsequently by gravitational instability processes.



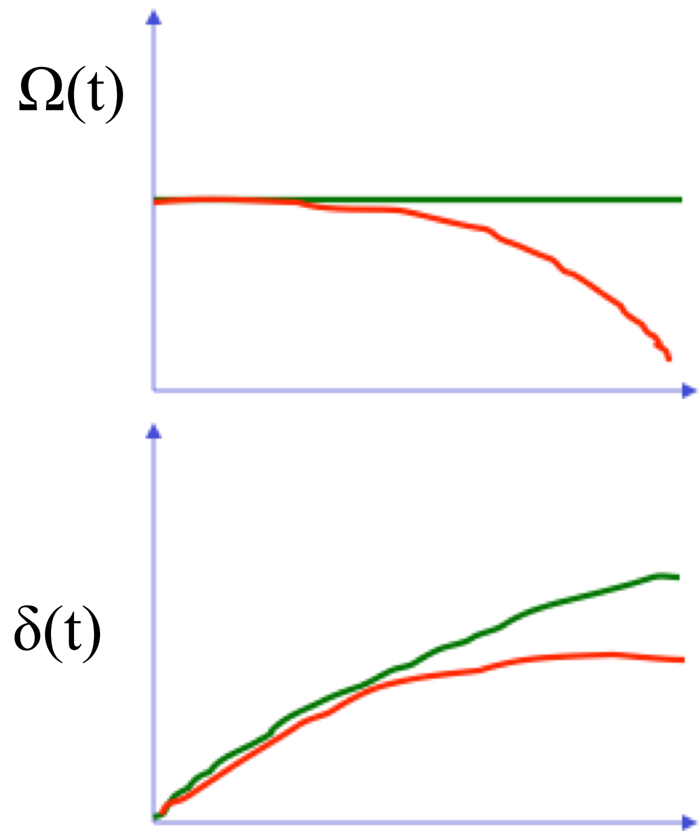
Growth of Density Fluctuations

- **Birkhoff‘s Theorem:** the matter inside a homogeneous sphere in a surrounding homogeneous Universe is not subjected to outside forces, except for tidal forces, which we can neglect here as second order effects. The matter density inside the sphere is evolving like a mini-Universe with the same density and expansion parameters.
- Each part of the Universe evolves approximately like a homogeneous Universe with the same mean density.
- Application to an inhomogeneous Universe :



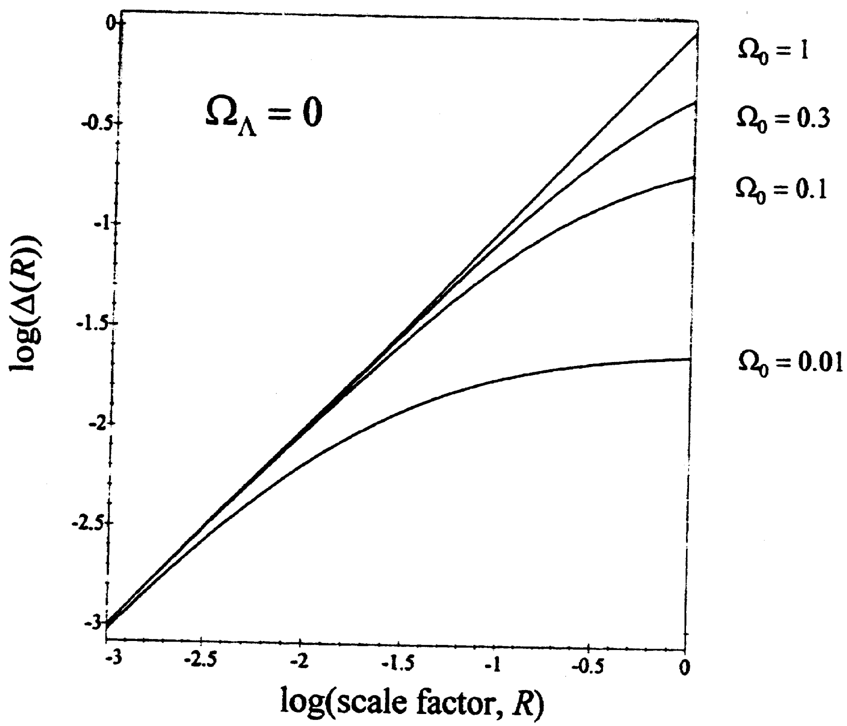
Structure Growth in a Critical Density and Open (Empty) Universe

- For small negative deviations of Ω from 1 the parameter evolves increasingly rapidly to small values.
- When Ω becomes significantly smaller than 1 the growth of fluctuations is stopped – the fluctuation spectrum is frozen.

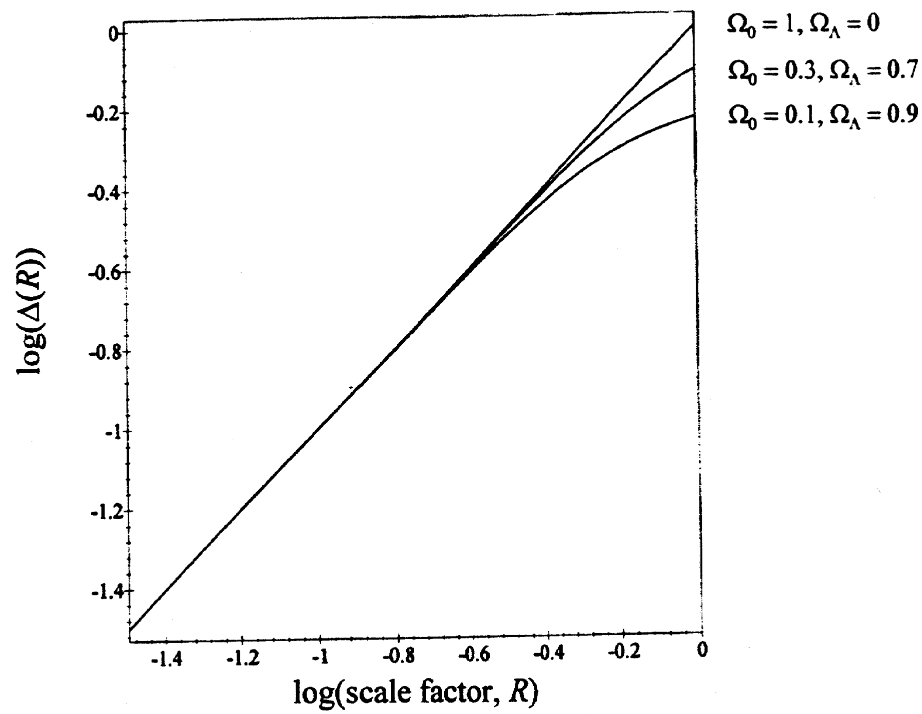


Growth of Density Fluctuations in Different Cosmological Models

Friedman-Lemaitre-Models

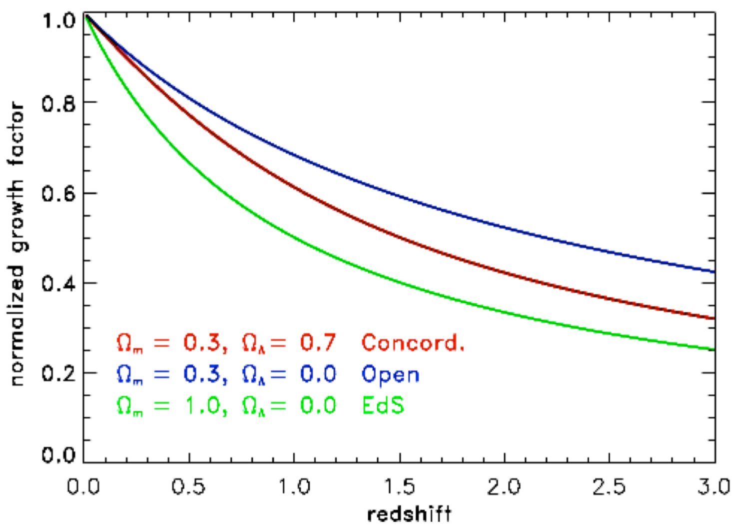


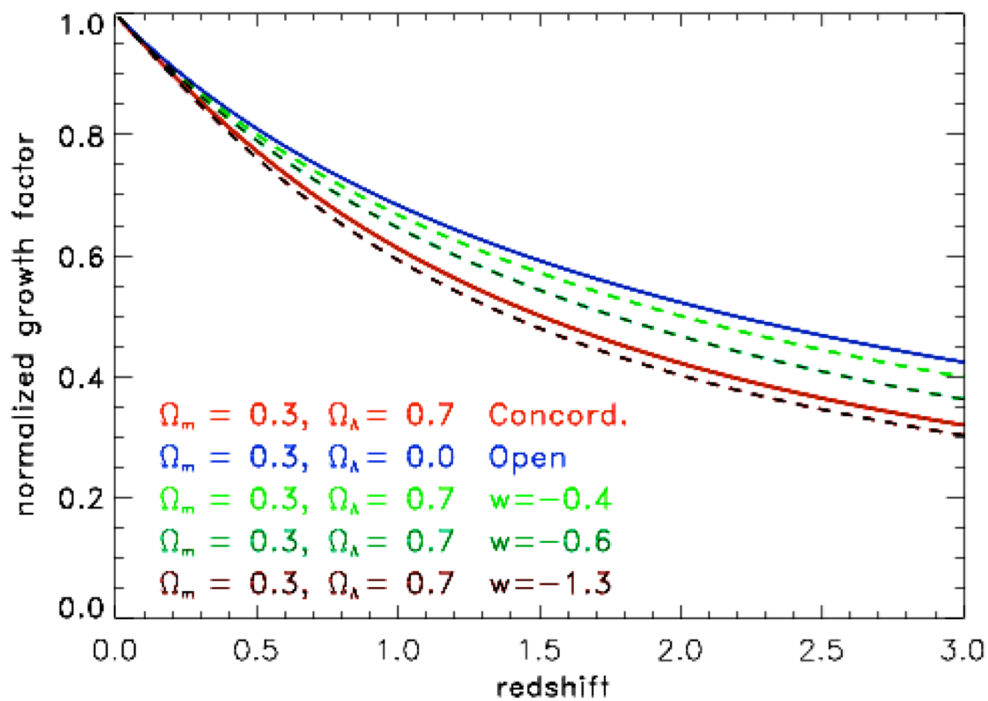
Λ-Models



Examples for the Growth of Structure in Different Cosmological Models including DE

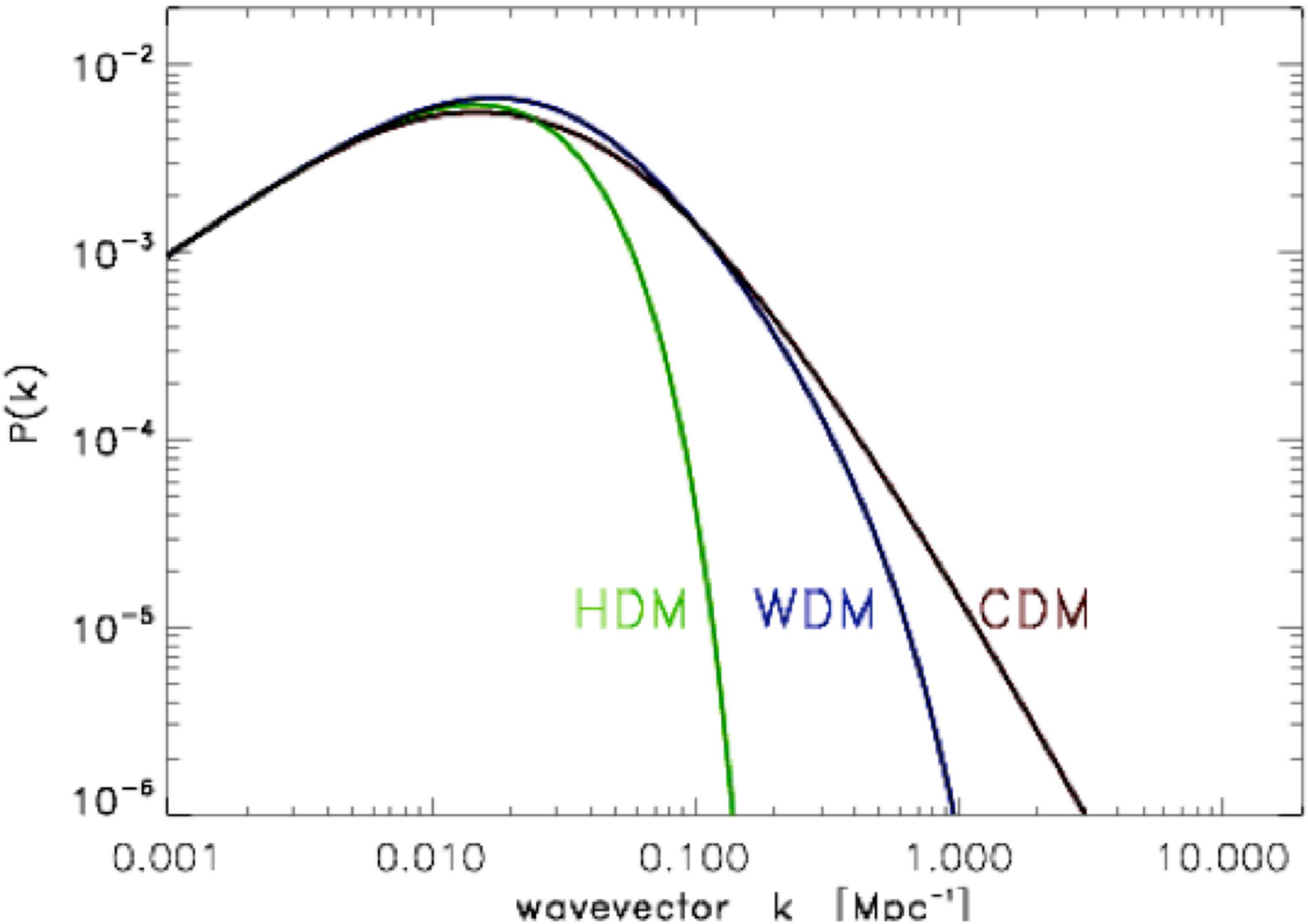
Density fluctuation growth:





- The more negative **w,** the stronger the evolution, that is, less clusters at high redshift for given normalization at present.

Power Spectra for Different Cosmologies



Relation Between P(k) and the Mean Amplitude of the Fluctuations as a Function of Scale (R)

Let the variance be $\Delta = \frac{\delta\rho}{\rho}$ $and\ P(k) \sim k^n$ we can then compute some statistics over a window function *W(kR):*

$\Delta^2(R) = \frac{1}{2\pi^2} \int k^{n+2}\, W(kR)^2 dk$ and so:

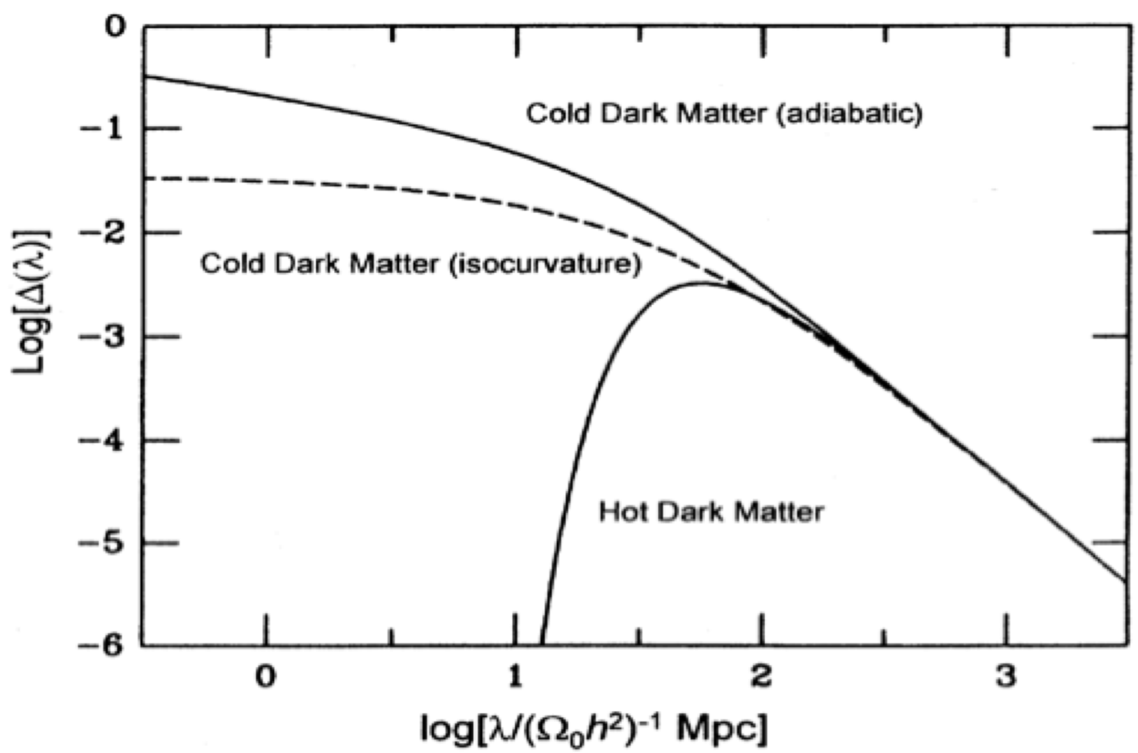
$\Delta^2(R) = \frac{1}{2\pi^2} R^{-(n+3)} \int R k^{n+2}\, W(kR)^2 dRk$ or:

$\Delta^2(R) = \frac{1}{2\pi^2} R^{-(n+3)} \int x^{n+2}\, W(x)^2 dx$

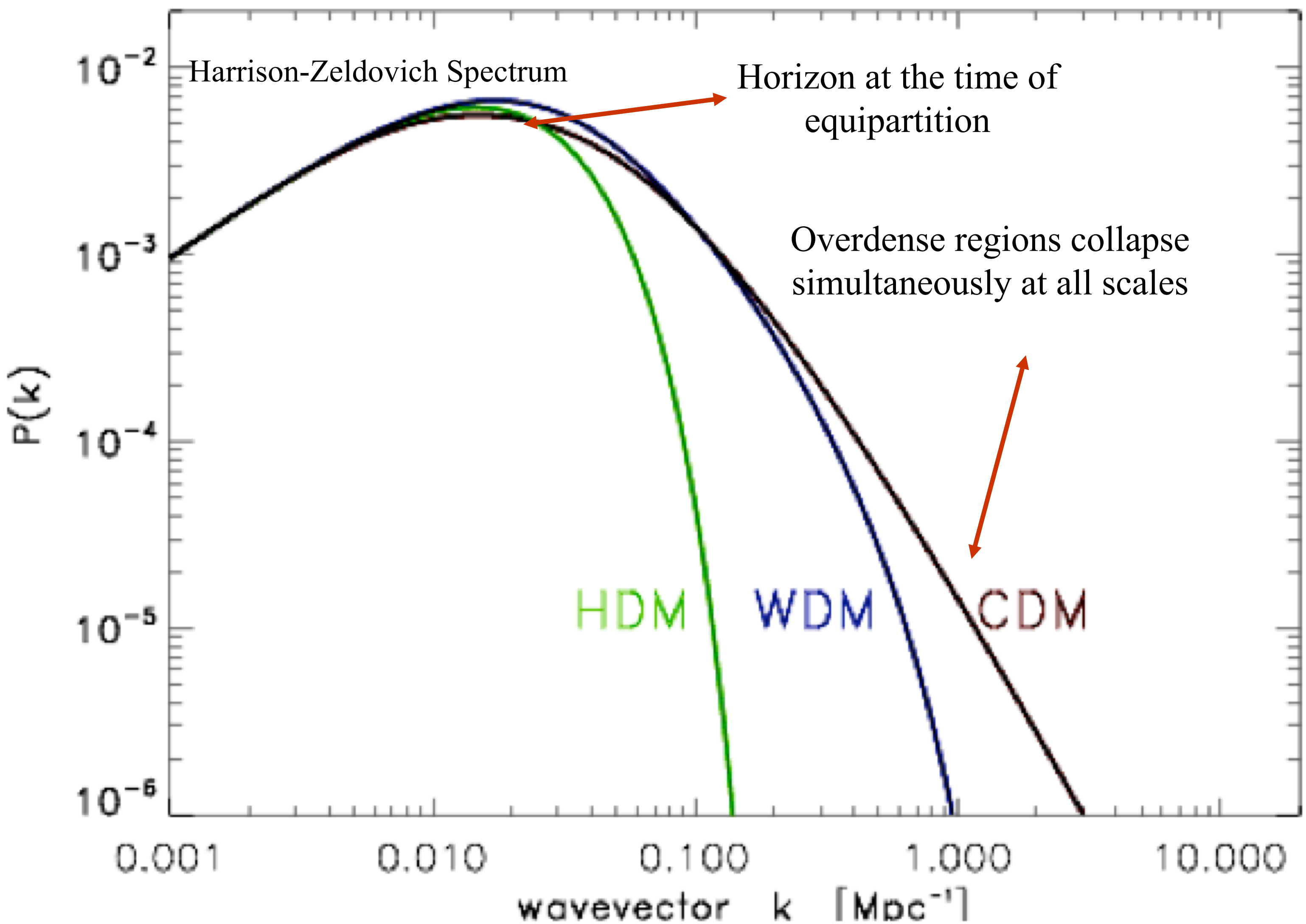
In which case: $\Delta \sim R^{-(n+3)/2}$ and $\int_0^\infty x^{n+2}\, W(x)^2 dx = \text{const.}$

Popular window functions are Gaussian or top-hat.

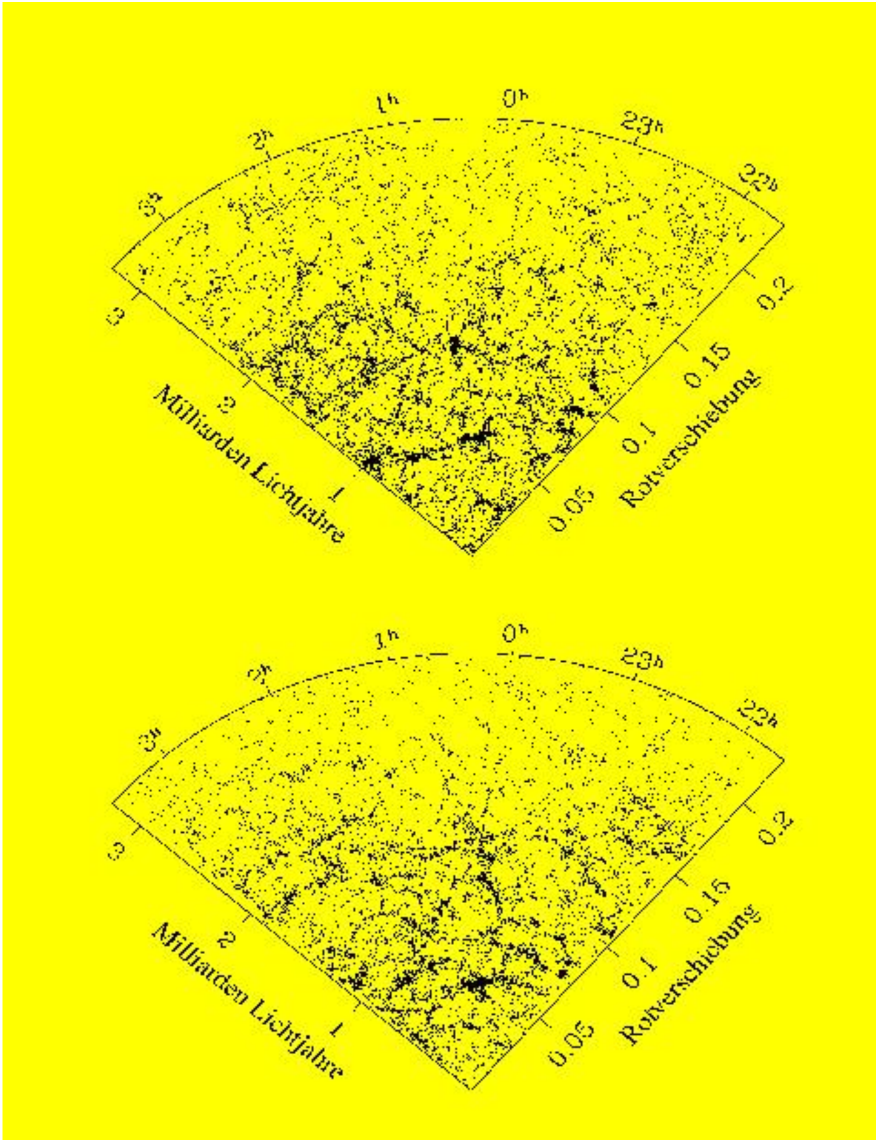
Variance of the Density Fluctuations as Function of the Wavelength for Different Cosmologies



Power Spectra for Dark Matter Scenarios



Example: 2dF-Galaxy Redshift Survey



Redshifts of > 100,000 galaxies

CDM-Simulations of this Survey