Astr 5465 Mar. 11, 2020
Fourier Techniques in Astronomy

- **Background**
  - Fourier Theorm: any function can be expressed as an infinite sum of basis vectors: sines & cosines
  - Fidelity of the Model is Better the More Frequencies are Used
    - Fourier Transform of a Function (or Signal) is a Series of Amplitudes for the Basis Vectors
    - See Bracewell for Examples of Simple Functions and Their Fourier Transforms
  - Pixelated Data Segment Contains Natural Low and High Frequency Cut-offs
    - Sampling Theorm Says There is No Information on Scales Smaller Than 2 pixels (highest frequency)
    - If the Data Segment (or Stream) is Limited Its Extent Limits the Lowest Frequency Sampled
  - FFT: Fast Fourier Transform for Digital Data
    - See numpy.fft.fft and Various Examples via Googling
- **Astronomical Spectroscopic Data**
  - Spectra & Spectral Lines Can be Modeled via a Set of Sines & Cosines
  - Finite Resolution of the Instrument Blurs the "True" Signal
  - If Spectrum is Resampled in Log Wavelength First a Shift Corresponds to a Velocity
  - Time Series of Spectra Can Be used to Measure Velocity Shift of Binary Star
  - Fourier Methods Use all the Data as Opposed to Measuring Position of a Single Spectroscopic Line
  - Velocity Dispersion (in say a galaxy) Acts as a Smoothing or Blurring of an Otherwise Sharp Spectrum
- **Astronomical Imaging Data**
  - Concept Can be Expanded to 2-d Imaging Data
    - Seeing or Finite Resolution of the Telescope Acts to Blur Astronomical Images
    - Information is Still Present and Can (in principle) be Recovered
    - Example: a Single Star Can be Used as a Reference to Recover Signal of Close Binary Star

Consider an Astronomical Spectrum

- Consider Example Spectrum (B&W) & Its Fourier Transform
- Spectrum Can Be Approximated with a Series of Sines & Cosines
- Finite Resolution of the Instrument & Discrete Pixel Sampling Results in a Cutoff Frequency (limiting information at smallest scales, i.e., highest frequencies)
- Real Data Has Noise (photon noise from signal, read noise of detector, etc)
- Noise is Present at the Highest Frequencies (pixel-to-pixel for digital data)

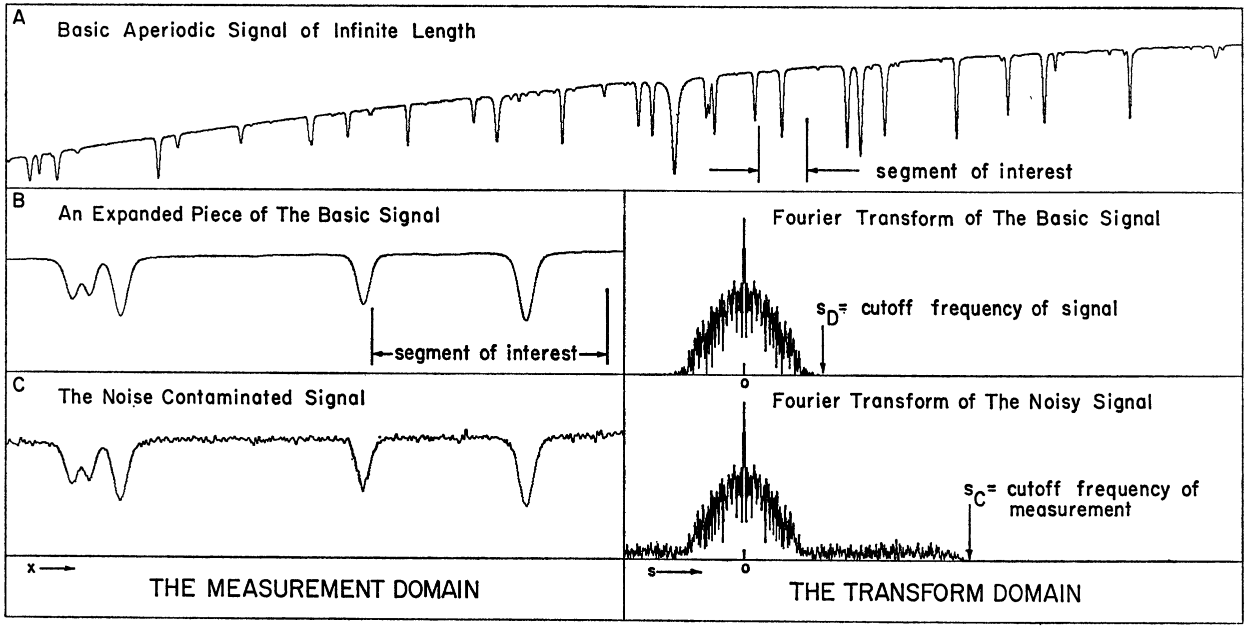

Fig. 1. Illustrations of the basic continuous signal, the noise-contaminated signal, and their transforms

Example of Ideal Line Profile

- True Spectrum is Convolved (smoothed) by the Instrument's Resolution (Apparatus)
- Since a Convolution in Real Space is a Multiplication in Fourier Space
  - Record Apparatus Function (e.g., comparison lines)
  - Divide it Into Data in Fourier Space
  - Inverse Transform to Recover Restored Profile
- Velocity Field of Elliptical Galaxies Acts Like an Apparatus (blurring) Function to the Spectrum of a Giant Star
  - Dividing the Spectrum by a Giant Star's Spectrum in Fourier Space
  - Transforming Back Recovers the Velocity Field (blurring function)
  - Fit this with Gaussian for Velocity and Velocity Dispersion

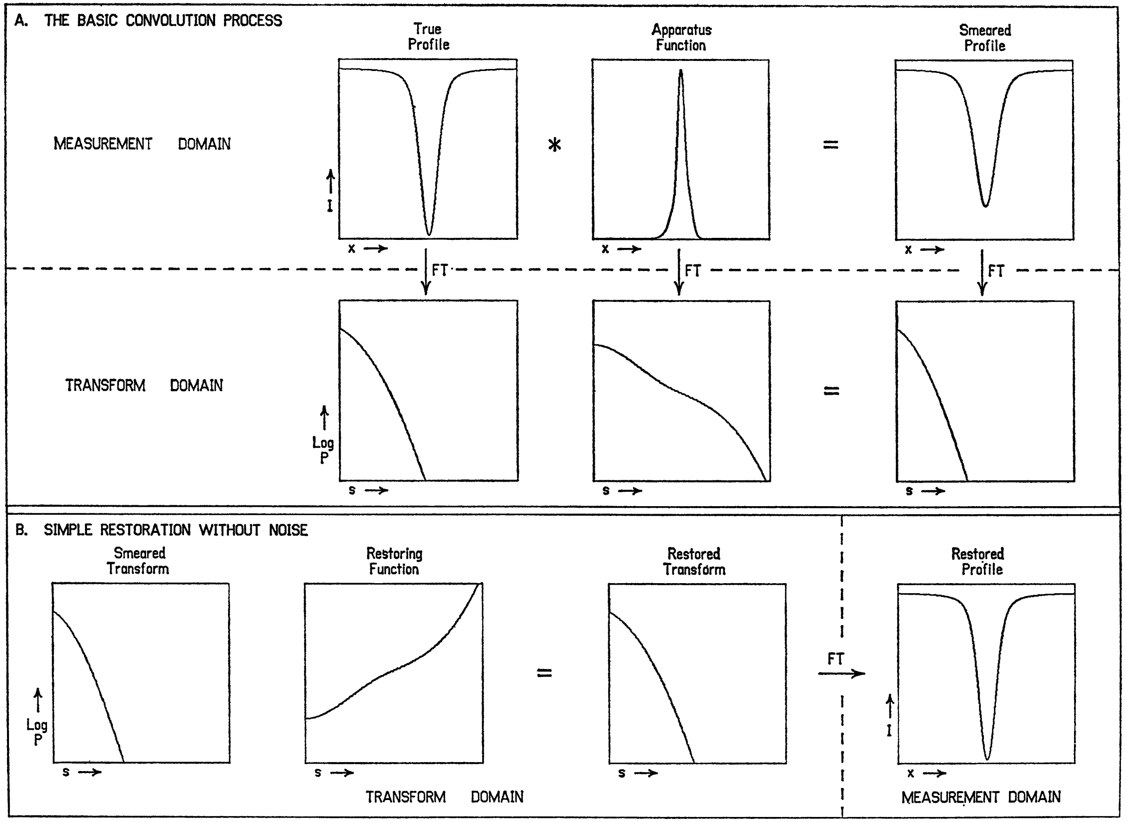

Fig. 12. Illustrations of (A) the convolution process as seen in both domains and (B) the simple Fourier restoration in the absence of noise

What About Real Spectrum with Noise?

- Fourier Transforms Amplify Noise into all Pixels
  - Note Frequency Cutoff in
  - Pure Data
  - Random Noise Means Noise at all Frequencies
  - “White Noise” Means Equal Power at all Frequencies
  - Fourier Transform Shows ”Noise Signal” Extending to High Frequencies
  - Inverse Transform Distributes this Noise into all Spatial Scales!

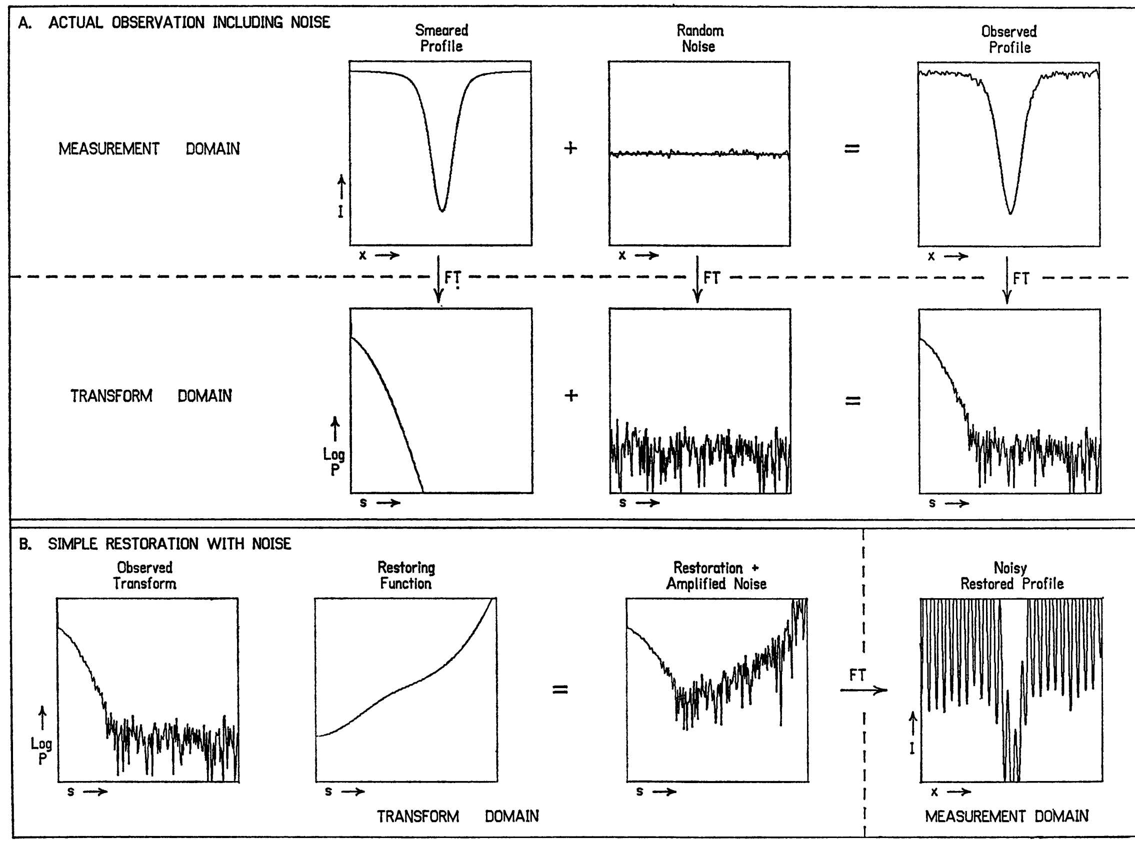

Fig. 13. Illustrations of (A) the effect of noise on the observed profile and its transform, and (B) the disastrous result of noise amplification in a simple restoration

Solution is Noise Filtering

- Instrumental Profile and Pixel Sampling (sampling theorm) Provides High Frequency Cutoff
- If we Construct a Soft-edged Filter We can Cutoff Some of the Noise in Fourier Space
- Distinction Between Signal & Noise is Most Obvious in a Power Spectrum
- The Inverse Transform No Longer Magnifies this Noise Since the Highest Frequencies are Filtered Out
- Technique Cannot Filter Out Low Frequency Noise but this is Rare in Most Astronomical Data

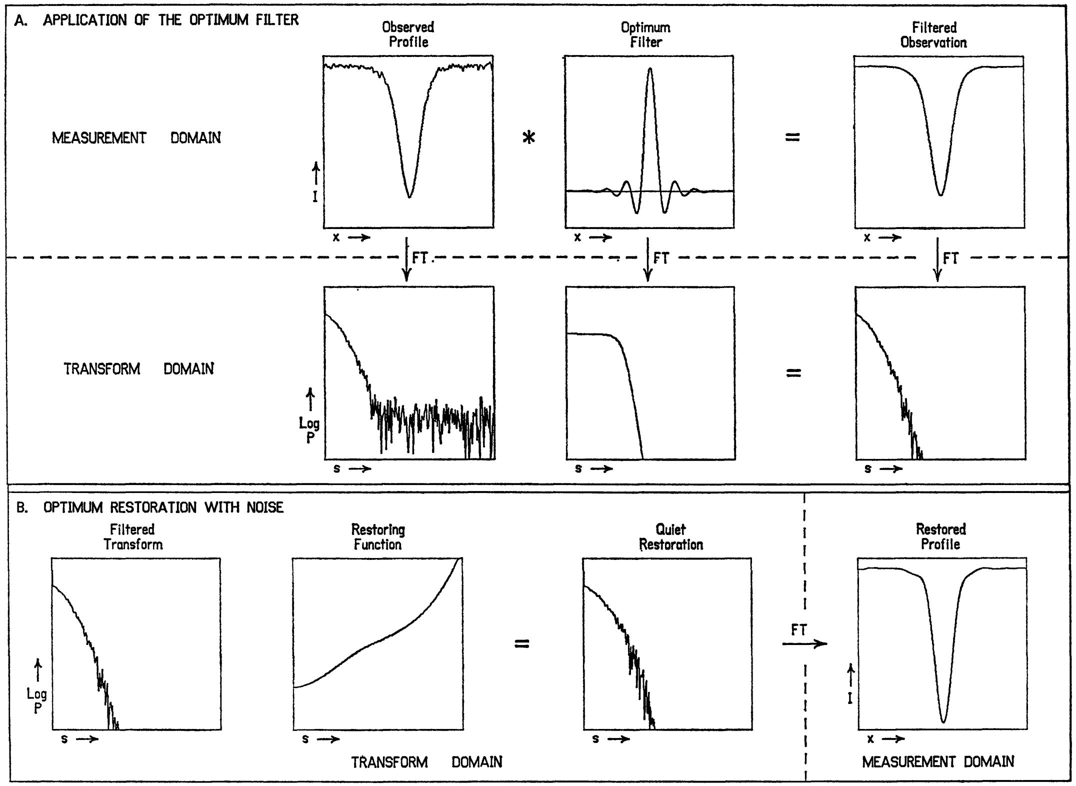

Fig. 14. Illustrations of (A) the noise reduction provided by the optimum filter, and (B) the resulting quiet restoration

References

- Fourier Transforms in Astronomy: Brault & White 1971, AA, 13, 169
- The Fourier Transform and its Applications: Bracewell, 2000, McGraw Hill
- The Observation & Analysis of Stellar Photospheres, Gray 2005, Chapter 2, Cambridge Univ. Press