ASTRO 1050
Measuring the Earth

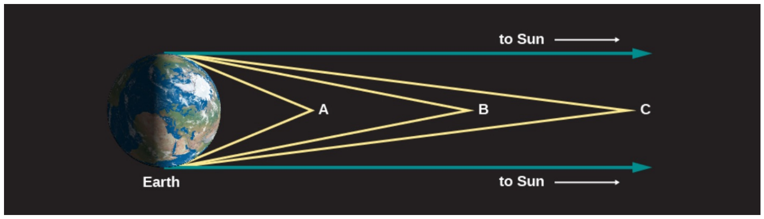
ABSTRACT

In this lab, we will attempt to measure some properties of the earth the way the first astronomers did to get a feel for how we first learned some of the things we know now. First, we will measure the radius of the earth like Eratosthenes, by using the difference in shadow length at different positions on the surface of the earth. Then we will use Newton's law of gravity to determine the mass of the earth by measuring the speed at which objects fall. As you work through this lab, make sure you take time to understand why these measurements work.

Measuring the Radius of the Earth

Eratosthenes was able to measure the radius of the earth using the difference in measurements of shadow length at two locations directly north-south of each other on the same day. To use this method, you first must accept that the earth is round. This can be proven by watching how ships disappear over the horizon, or just by examining this image taken by the international space station, where we can see that the shape of the earth is in fact spherical.

We also must use our powers of observations to tell that the sun is far enough away that all rays from the sun hit the earth in parallel lines. This is shown in the figure below. If the sun was at points A, B, or C, this method would not work. But, since the sun is 1.496×10^{11} meters away, all rays are essentially parallel and this image is correct.



With these ideas in mind, we can start to get to the actual measurement. As we have stated that all of the sun's rays hit the earth in parallel lines, the difference in angle at which they hit the surface of the earth is only determined by the curvature of the earth and your location along that curve. Eratosthenes knew that no shadow would be cast by the sun's rays in Syene on a specific day, as the rays fell directly into the well. He was then able to measure the shadow of a column further north and determine the difference in angle and use that to find what fraction of the earth's circumference he had moved across between measurements. The image below shows this as well.

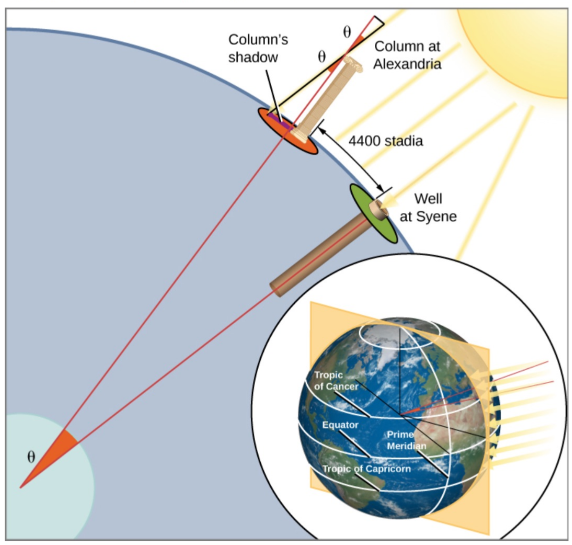


Figure 2.11. How Eratosthenes Measured the Size of Earth. Eratosthenes measured the size of Earth by observing the angle at which the Sun's rays hit our planet's surface. The Sun's rays come in parallel, but because Earth's surface curves, a ray at Syene comes straight down whereas a ray at Alexandria makes an angle of 7° with the vertical. That means, in effect, that at Alexandria, Earth's surface has curved away from Syene by 7° of 360°, or 1/50 of a full circle. Thus, the distance between the two cities must be 1/50 the circumference of Earth. (credit: modification of work by NOAA Ocean Service Education)

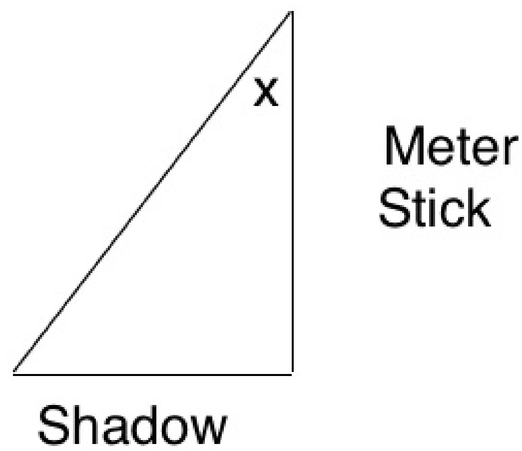
So, if we measured the angle at the north pole and at the equator, we should see a 90°difference, and we would know we had travelled $\frac{90}{360}$ or 1/4 of the way around the earth. Then, if we knew the distance we travelled, we could use the formula for the circumference of a sphere ($2\pi R$) to solve for R with the knowledge that the distance we travelled is 1/4 of the total circumference.

Unfortunately we do not have the ability to travel that far. But, we are lucky enough to have a partner in Fort Collins who is going to measure the shadow cast by a meter stick there, so we should be able to do this! We can then complete our own measurement here, find the difference, and use Eratosthenes' method to measure the radius of the earth. Fort Collins is 50 miles south of Laramie, so once you have both these measurements you will find the radius of the earth!

Length of Shadow at Fort Collins: ______________________

Length of Shadow at Laramie: ______________________

We can find the difference in angle using the Pythagorean theorem, as we have two right triangles with two known sides each, as shown in the figure below. Determine what the angle x is for both triangles. (*Hint*: Remember, $\sin x =$opposite/hypotenuse, $\cos x =$ adjacent/hypotenuse and $\tan x =$ opposite/adjacent).



Angle of sun's shadow at Fort Collins: ______________________

Angle of sun's shadow at Laramie: ______________________

Difference in angles: ______________________

Finally, we can put all this together to measure the radius of the earth! Use the following formula and the data you found to solve for R.

$$\frac{\text{Difference in angles}}{360} = \frac{\text{Laramie-Fort Collins distance}}{2\pi R}$$

Radius of Earth: ______________________________

The actual radius of the earth is 6.378×10^6 meters. Use the formula below to determine your percent error (Don't worry if it's large, I'm not grading on accuracy here).

$$\text{percent error} = \frac{\text{actual - measured}}{\text{actual}} \times 100\%$$

What could you do to improve the accuracy of this measurement?

Measuring the Mass of the Earth

With this value, we can now also determine the mass of the earth! To do this, we will use Newton's law of gravity. This says that the force due to gravity is:

$$F = \frac{GMm}{R^2}$$

Here G is the universal gravitational constant, 6.67×10^{11} $m^3/kg\ s^2$, M and m are the masses of the two objects, and R is the distance between the two objects. We can also write this formula as:

$$F = gm$$

where g is the acceleration due to gravity (how fast an object speeds up as it falls) and m is the mass of the falling object. So, if we can measure g, we can use the radius we found earlier to determine M, the mass of the earth!

First, write a formula relating g and M using the two equations for the force of gravity.

Now we need to measure g. To do this, we need to determine how long it takes for an object to drop a certain distance. Take one of the tennis balls and stop watches to the atrium and work with your group to determine how long it takes for the tennis ball to fall from either the first or second floor. You will also need to determine how high this is, either by using one of the long measuring tapes or taking a picture while holding a meter stick and using this as a scale to find the total height (Talk to Jessica if you are confused).

Time for ball to drop: ________________________

Distance ball dropped: ________________________

We are going to need a little physics to determine the acceleration now. When an object starts at rest, the acceleration can be found with the formula:

$$\Delta x = \frac{1}{2}at^2$$

or

$$g = \frac{2\Delta x}{t^2}$$

where Δx is the distance the ball dropped and t is the time it took for the ball to drop. Use your measurements to find g.

acceleration due to gravity (g): ______________________________

You are almost there! Now just plug this into your earlier equation relating g and M to determine the mass of the earth.

Mass of Earth: ______________________________

Congratulations! You just determined the mass and radius of the earth on your own! Pat yourself on the back for a lab well done.